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ing of the mid-points of the segments, P_1P_2, P_2P_3, ..., P_nP_1. This set is entirely arbitrary if n is odd, but not if n is even; the characteristics stated in 9 apply almost literally to these special sets of points of the latter type.

Columbia University, New York, *January 15, 1903.*

ANALOG OF SYLVESTER'S DIALYTIC METHOD OF ELIMINATION.

By DR. SAUL EPSTEEN.

If the two equations

$$\begin{aligned} a_0x^3+a_1x^2+a_2x+a_3&=0, \\ b_0x^2+b_1x+b_2&=0, \end{aligned}$$

have a root x_1 in common, we can write

$$\begin{aligned} a_0x_1^4+a_1x_1^3+a_2x_1^2+a_3x_1 \qquad &\equiv 0 \\ a_0x_1^3+a_1x_1^2+a_2x_1+a_3 &\equiv 0 \\ b_0x_1^4+b_1x_1^3+b_2x_1^2 \qquad &\equiv 0 \\ b_0x_1^3+b_1x_1^2+b_2x_1 \qquad &\equiv 0 \\ b_0x_1^2+b_1x_1+b_2 &\equiv 0. \end{aligned}$$

Eliminating x_1^4, x_1^3, x_1^2, x_1, 1, we obtain

$$\begin{vmatrix} a_0 & a_1 & a_2 & a_3 & 0 \\ 0 & a_0 & a_1 & a_2 & a_3 \\ b_0 & b_1 & b_2 & 0 & 0 \\ 0 & b_0 & b_1 & b_3 & b_1 \\ 0 & 0 & b_0 & b_1 & b_2 \end{vmatrix} = 0.$$

This is the well known dialytic method of elimination of Sylvester.

We can deal with linear differential equations in exactly the same manner. Suppose there are given the equations

$$(1) \qquad \alpha_0(x)\frac{d^3y}{dx^2}+\alpha_1(x)\frac{d^2y}{dx^2}+\alpha_2(x)\frac{dy}{dx}+\alpha_3(x).y=0,$$

$$(2) \qquad \beta_0(x)\frac{d^2y}{dx^2}+\beta_1(x)\frac{dy}{dx}+\beta_2(x).y=0.$$

If these equations have an integral y_1 in common we can, by differentiation, write

$$\alpha_0\frac{d^4y_1}{dx_4}+(\alpha_0'+\alpha_1)\frac{d^3y_1}{dx^3}+(\alpha_1'+\alpha_2)\frac{d^2y_1}{dx^2}+(\alpha_2'+\alpha_3)\frac{dy_1}{dx}+\alpha_3'y_1\equiv 0.$$

$$\alpha_0\frac{d^3y_1}{dx^3}+\qquad\alpha_1\frac{d^2y_4}{dx^2}+\qquad\alpha_2\frac{dy_1}{dx}+\alpha_8\,y_1\equiv 0.$$

$$\beta_0\frac{d^4y_1}{dx^4}+(2\beta_0'+\beta_1)\frac{d_8y_1}{dx^3}+(\beta_0''+2\beta_1'+\beta_2)\frac{d^2y_1}{dx^2}+(\beta_1''+2\beta_2')\frac{dy_1}{dx}+\beta_2''y_1\equiv 0.$$

$$\beta_0\frac{d^2y_1}{dx^2}+\qquad(\beta_0'+\beta_1)\frac{d^2y_1}{dx^2}+\qquad(\beta_1'+\beta_2)\frac{dy_1}{dx}+\beta_2'\,y_1\equiv 0.$$

$$\beta_0\frac{d^2y_1}{dx^2}+\qquad\beta_1\frac{dy_1}{dx}+\beta_2y_1\equiv 0.$$

Whence, eliminating $\frac{d^4y_1}{dx^4}$, $\frac{d^3y_1}{dx^3}$, $\frac{d^2y_1}{dx^2}$, $\frac{dy_1}{dx}$, y_1, we find that

$$\begin{vmatrix} \alpha_0 & (\alpha_0'+\alpha_1) & (\alpha_1'+\alpha_2) & (\alpha_2'+\alpha_3) & \alpha_3' \\ 0 & \alpha_0 & \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_0 & (2\beta_0'+\beta_1) & (\beta_0''+2\beta_1'+\beta_2) & (\beta_1''+2\beta_2') & \beta_2'' \\ 0 & \beta_0 & (\beta_0'+\beta_1) & (\beta_1'+\beta_2) & \beta_2' \\ 0 & 0 & \beta_0 & \beta_1 & \beta_2 \end{vmatrix}=0,$$

for all values of x.

The generalization to equations of order m and n is of course easy.

This theorem is not to be found in any of the standard treatises or text-books. Indeed it is rather surprising that Schlesinger's *Handbuch der Theorie der linearen Differentialgleichungen*, which is such a complete treatise on linear differential equations, does not contain it. The theorem was proved by Professor von Escherisch in the *Denkschriften der Wiener Akademie*, Vol. 46. However, these *Denkschriften* have but a very small circulation in America and the volumes preceeding the 64th seem to be particularly rare. Even such complete collections as the University of Chicago Library and the John Crerar Library (Chicago) do not have them. Possibly this accounts for the oblivion in which Professor von Escherisch's theorem has hitherto rested. Personally I have never been able to get a copy of Vol. 46 of the *Denkschriften der Wiener Akademie* to consult the paper at first hand and know of it only through the reference given by Heffter in Crelle's *Journal*, Vol. 116.

THE UNIVERSITY OF CHICAGO, *December, 1902.*